Statistics

Summer 2021

Lecture 7

Quick Review

Given $P(A) = .24$

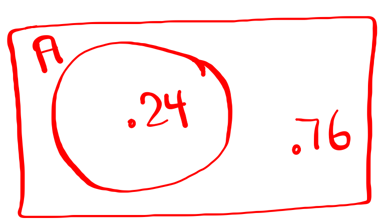
1) write $P(A)$ in reduced fraction

$.24$ $\boxed{\text{Math}}$ $\boxed{1!}$ $\boxed{\text{Enter}}$ $\frac{6}{25}$

2) find $P(\bar{A})$ in %.

$P(\bar{A}) = 1 - P(A) = .76 = .76(100)\% = \boxed{76\%}$

3) Draw Venn Diagram

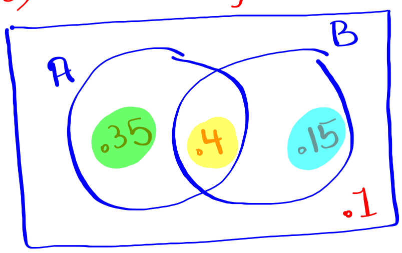


Given $P(A) = .75$, $P(B) = .55$, $P(A \text{ and } B) = .4$

1) $P(\overline{A}) = 1 - P(A) = \boxed{.25}$

2) $P(\overline{B}) = 1 - P(B) = \boxed{.45}$

3) Venn Diagram



4) $P(A \text{ or } B)$
$= P(A) + P(B) - P(A \text{ and } B)$
$= .75 + .55 - .4 = \boxed{.9}$

5) $P(A \text{ only} \text{ or } B \text{ only})$
$= .35 + .15 = \boxed{.5}$

6) $P(\overline{A} \text{ and } \overline{B}) = P(\overline{A \text{ or } B})$
$= \boxed{.1}$

De Morgan's Law

7) $P(\overline{A} \text{ or } \overline{B}) = P(\overline{A \text{ and } B}) = \boxed{.6}$

A and B are disjointed events.

$P(A) = .15$, $P(B) = .65$

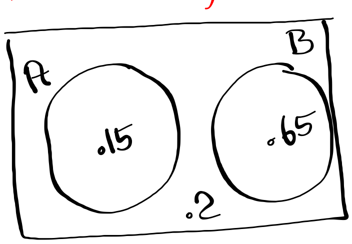
1) $P(\overline{A}) = 1 - P(A) = \boxed{.85}$

2) $P(\overline{B}) = 1 - P(B) = \boxed{.35}$

3) $P(A \text{ and } B) = \boxed{0}$

4) $P(A \text{ or } B)$
$= P(A) + P(B) - P(A \text{ and } B)$
$= .15 + .65 - 0$
$= \boxed{.8}$

5) Venn Diagram



6) $P(\overline{A} \text{ and } \overline{B})$

De Morgan's Law

$= P(\overline{A \text{ or } B})$

$= \boxed{.2}$

Multiplication Rule:

Keyword: AND

Multiple - Action event

$$P(\underbrace{A \text{ and } B}) = P(A) \cdot P(B|A)$$

A happens then B happens

given

A box has 3 Red and 2 Blue balls.

Randomly draw 2 balls, with replacement.

$$P(2 \text{ Red Balls}) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25} = \boxed{.36}$$

$$P(2 \text{ Blue Balls}) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} = \boxed{.16}$$

Repeat the last example without replacement

3 Red & 2 Blue

$$P(2 \text{ Reds}) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \boxed{.3}$$

$$P(2 \text{ Blue}) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} = \boxed{.1}$$

A piggy bank has 3 dimes & 7 Nickels.
Randomly Select 2 Coins. with replacement

$\left.\begin{array}{l} DD \\ DN \\ ND \\ NN \end{array}\right\}$ Sample Space

DD → 20¢

DN, ND → 15¢

NN → 10¢

$P(20¢) = P(DD) = \frac{3}{10} \cdot \frac{3}{10} = \frac{9}{100} = \boxed{.09}$

$P(15¢) = P(DN \text{ or } ND) = \frac{3}{10} \cdot \frac{7}{10} + \frac{7}{10} \cdot \frac{3}{10} = \frac{42}{100} = \boxed{.42}$

$P(10¢) = P(NN) = \frac{7}{10} \cdot \frac{7}{10} = \frac{49}{100} = \boxed{.49}$

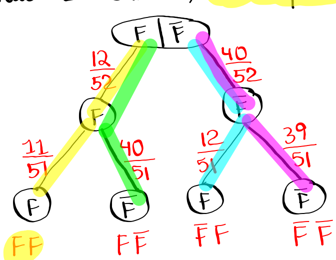
Total ¢	P(Total ¢)
20	.09
15	.42
10	.49

Total ¢ → L1
P(Total ¢) → L2
1-Var Stats
$\bar{x} = 13$ S = Blank

Total Prob.
↓
$\boxed{n = 1}$

Multiplication with Tree
Rule Diagram

A deck of cards has 52 cards, 12 face cards. Draw 2 cards, No replacement



$P(2 \text{ Face Cards}) = \frac{12}{52} \cdot \frac{11}{51} = \boxed{\frac{11}{221}}$

$P(1 \text{ Face}, 1 \overline{\text{Face}}) = \frac{12}{52} \cdot \frac{40}{51} + \frac{40}{52} \cdot \frac{12}{51} = \boxed{\frac{80}{221}}$

$P(0 \text{ Face Cards}) = \frac{40}{52} \cdot \frac{39}{51} = \boxed{\frac{10}{17}}$

Verify $\frac{11}{221} + \frac{80}{221} + \frac{10}{17} = \boxed{1}$ ✓

# Face	P(# Face)
2	11/221
1	80/221
0	10/17

Face → L1
P(# Face) → L2
1-Var Stats
$\bar{x} = .462$ S = Blank n = 1

Independent events:

outcome of one event does not change the prob. of next event.

flip a coin, New born babies, Select cards with replacement, ...

If A and B are independent events, then

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

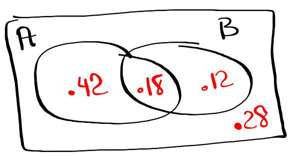
$P(A) = .6$, $P(B) = .3$, A & B are independent events.

$P(\bar{A}) = 1 - P(A) = \boxed{.4}$

$P(\bar{B}) = 1 - P(B) = \boxed{.7}$

$P(A \text{ and } B) = P(A) \cdot P(B) = (.6)(.3) = \boxed{.18}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = .6 + .3 - .18 = \boxed{.72}$



Dependent events

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

Given

Standard deck of playing cards:

52 cards, 26 Red, 12 face, 4 Aces.

Draw 3 cards, No replacement.

$$P(3 \text{ Red Cards}) = \frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50} = \boxed{\frac{2}{17}}$$

$$P(3 \text{ face Cards}) = \frac{12}{52} \cdot \frac{11}{51} \cdot \frac{10}{50} = \boxed{\frac{11}{1105}}$$

$$P(3 \text{ Aces}) = \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} = \boxed{\frac{1}{5525}}$$

Odds vs Probabilities

I tossed a coin 50 times, and recorded 20 Tails.

20 Tails : 30 $\overline{\text{Tails}}$ $\Rightarrow$ 2:3

odds in favor of landing tails.

I attempted 40 shots in a basketball court.

I made 12 and I missed 28.

12 Made   28 $\overline{\text{Make}}$ $\Rightarrow$ 3:7

odds in favor of making shots

odds in favor of event E are a:b

odds against event E are b:a

How to find odds when $P(E)$ is given:

$P(E) : P(\overline{E})$

Always reduce.

Given $P(E) = 2.5\%$

1) $P(E)$ in decimal $P(E) = .025$

2) $P(\overline{E}) = 1 - P(E) = .975$

$$\frac{P(E)}{P(\overline{E})} = \frac{.025}{.975} = \frac{1}{39} \Rightarrow \boxed{\text{odds are } 1:39}$$

Suppose $P(E) = .24$

1) $P(\overline{E}) = 1 - P(E) = .76$

2) odds in favor of event E. $\boxed{6:19}$

$$\frac{P(E)}{P(\overline{E})} = \frac{.24}{.76} = \frac{6}{19} \Rightarrow \boxed{6:19}$$

3) odds against event E. $\boxed{19:6}$

How to find $P(E)$ & $P(\bar{E})$ when odds for event E are $a:b$.

$P(E) = \frac{a}{a+b}$ $P(\bar{E}) = \frac{b}{a+b}$

odds for event E are $3:47$.

$P(E) = \frac{3}{3+47} = \frac{3}{50} = \boxed{.06}$ $P(\bar{E}) = \frac{47}{3+47} = \frac{47}{50} = \boxed{.94}$

12 Males & 38 Females

$P(\text{Select 1 Female}) = \frac{38}{50} = \boxed{.76}$

odds in favor of selecting female

38 Females 12 Females

$\Rightarrow 38:12 \Rightarrow \boxed{19:6}$

True

Odds in favor of event E

$$a : b$$

\$ bet   \$ Net return

Suppose odds are 3:400

\$3 bet → \$400 Net return

How much do I bet to make \$10,000 net return?

$$\frac{\$3 \text{ bet}}{\$400 \text{ Net}} = \frac{\$x \text{ bet}}{\$10{,}000 \text{ Net}}$$

$$\frac{3}{400} = \frac{x}{10000}$$

Cross-Multiply, Solve for x

$400x = 3(10000)$ $x = \frac{30000}{400}$

$$\boxed{x = 75}$$

\$75 bet

→ Net return \$10,000